coreform IGA
for Abaqus

Elastic spherical indenter sliding on a block

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1 Objective

Verify deformable-body, frictional, finite-sliding surface-to-surface contact. A hollow spherical indenter is first pressed into an elastic block and then translated across its upper surface. Normal force and maximum contact pressure are compared with the Hertz elastic solution, while tangential reaction is compared with the Coulomb sliding limit.

2 Geometry

The block occupies a $5 \times 2 \times 3$ inch rectangular domain. The indenter is the bottom half of a hollow sphere, cut through its center by a plane parallel to the x-z plane. It has an $R = 0.5$ inch outer radius and a 0.25 inch inner radius. The geometry and initial position are shown in Figure 1. Its initial center is at $(-1.5, 1.525, 0)$, leaving a 0.025 inch gap above the block's top surface at $y = 1$.

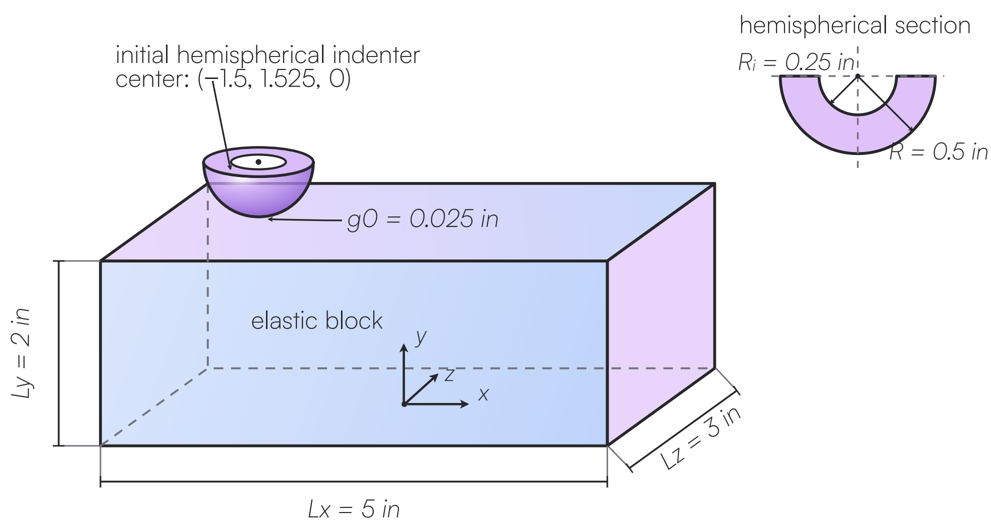


Figure 1: Initial contact-indenter geometry. The hollow lower-hemispherical indenter is centered above the left side of the elastic block; the small initial clearance is shown by a callout, and the inset identifies the indenter's inner and outer radii.

Table 1: Geometric parameters.

Quantity	Symbol	Value (in)
Block dimensions	$L_x \times L_y \times L_z$	$5 \times 2 \times 3$
Indenter outer radius	R	0.5
Indenter inner radius	R_i	0.25
Initial clearance	g_0	0.025

3 Material

Both bodies use geometrically non-linear isotropic, small-strain linear elasticity. The material model is intentionally kept elastic even though the analytical contact pressure is far above the yield stress of ordinary steel; this study is a mathematical verification of the elastic contact implementation rather than a physical steel-indentation experiment.

Table 2: Linear-elastic material properties.

Body	Young's modulus (psi)	Poisson's ratio
Block	$2.9e+07$	0.3
Indenter	$3e+07$	0.29

4 Loading & boundary conditions

The lower face of the block is pinned. During $0 \leq t \leq 0.1$, the upper planar face of the indenter receives a prescribed vertical displacement of -0.125 in. During $0.1 \leq t \leq 1$, that displacement is held while the face translates 4.25 in in the positive x direction. The contact pair uses finite sliding, surface-to-surface enforcement, and isotropic penalty Coulomb friction with $\mu = 0.3$ and an elastic-slip fraction of 0.005 [1].

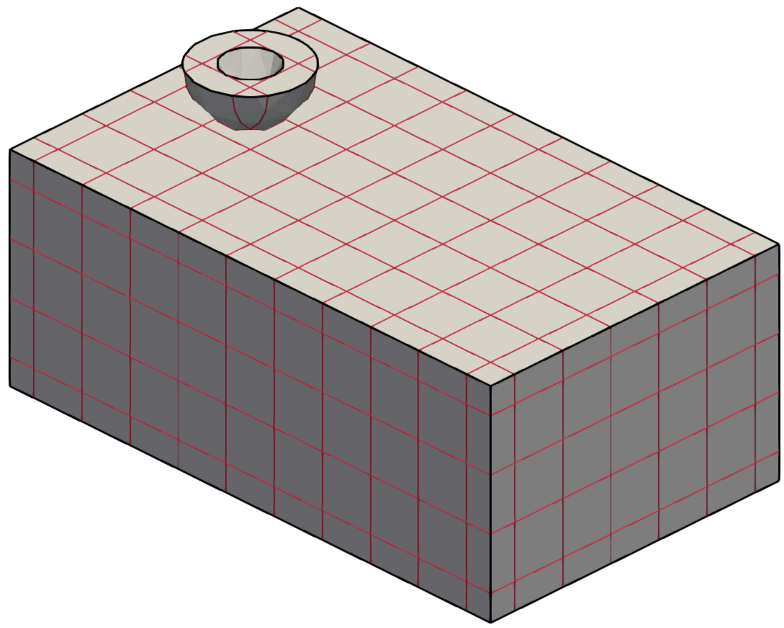
5 Mesh & discretization

The mesh study contains native Abaqus and Coreform IGA cases at refinement levels $h_0 = 0.5$ in, $h_1 = 0.25$ in, and $h_2 = 0.125$ in. The Abaqus results use C3D8R elements. The Coreform IGA results use quadratic elements with C^1 continuity.

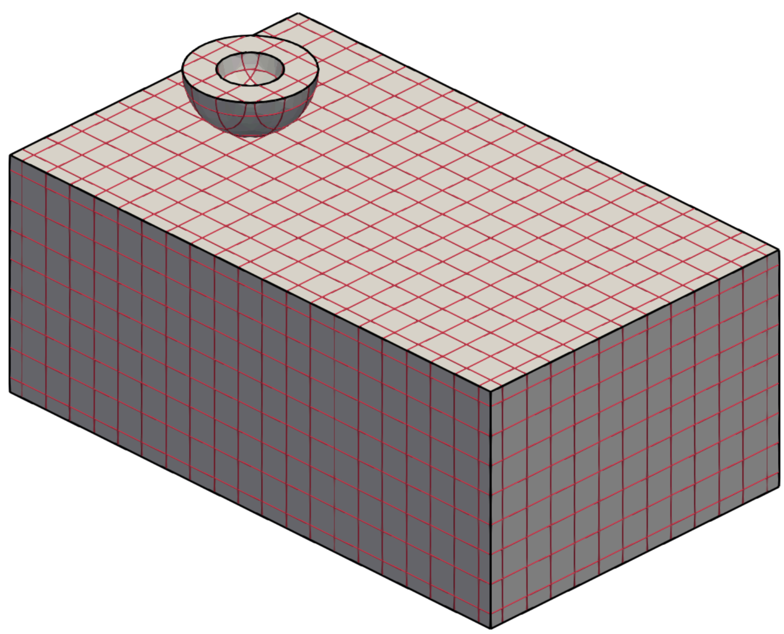
The predicted Hertz contact diameter at full indentation is approximately 0.447 in. It is slightly smaller than the h_0 mesh size, while h_1 and h_2 progressively resolve the patch. This makes maximum pressure a more demanding refinement quantity than the integrated reaction forces.

The tabs below show the Coreform IGA discretizations at each refinement level. Trimmed Bézier element edges are shown in carnation and CAD edges in black.

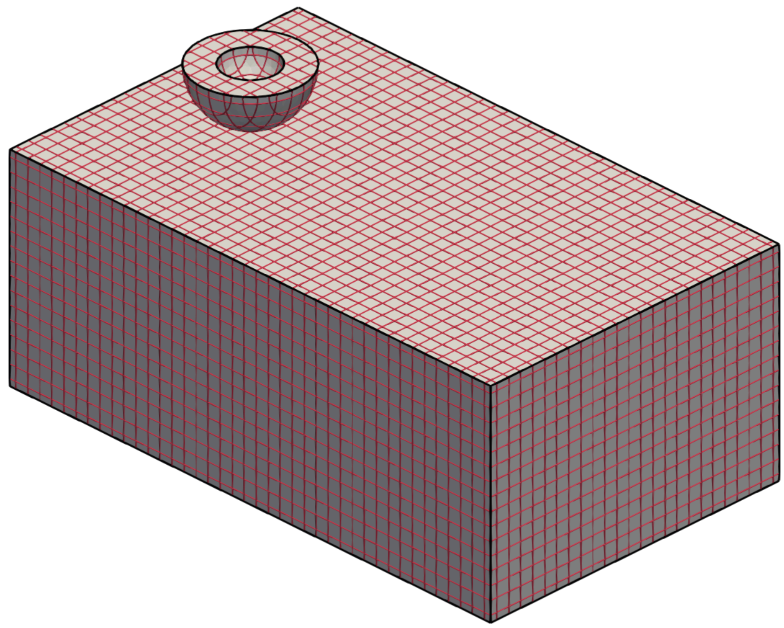
5.0.a $h_0 = 0.5$ **in**



5.0.b $h_1 = 0.25$ **in**



5.0.c $h_2 = 0.125$ in



6 Reference & accepted solutions

For a spherical surface contacting an elastic half-space, the Hertz solution uses the reduced modulus [2], [3]

$$\frac{1}{E^*} = \frac{1-\nu_b^2}{E_b} + \frac{1-\nu_i^2}{E_i}.$$

With indentation δ, sphere radius R, and contact radius a, the accepted normal quantities are

$$a = \sqrt{R\delta}, \qquad F_N = \frac{4}{3}E^*\sqrt{R}\,\delta^{3/2}, \qquad p_0 = \frac{2E^*}{\pi}\sqrt{\frac{\delta}{R}},$$

where a is the contact radius, F_N is the normal force, and p_0 is the maximum pressure. The initial clearance is subtracted from the imposed motion, giving

$$\delta(t) = \max(1.25t - 0.025, 0), \quad 0 \le t \le 0.1,$$

and $\delta = 0.1$ in. thereafter while the contact patch remains on the planar top surface. During gross sliding the tangential reference is $F_T = \mu F_N$. The sliding comparison uses $0.15 \le t \le 0.70$; later motion approaches the finite block edge and is outside the half-space solution.

Table 3: Hertz-Coulomb accepted quantities at full indentation.

Quantity	Accepted value
Reduced modulus E^*	$1.615266e+07$ psi
Full indentation δ	0.1 in
Contact radius a	0.223607 in
Normal force F_N	481579.3 lbf
Gross-sliding force F_T	144473.8 lbf
Maximum pressure p_0	4598743 psi

7 Results

7.1 Quantities of interest

The test enforces pointwise relative errors at the end of indentation and relative RMS errors over the gross-sliding comparison window where RMS denotes root mean square. For computed samples q_i and corresponding analytical values q_i^*, the relative RMS error reported here is

$$e_{\mathrm{RMS}} = \frac{\sqrt{\sum_{i=1}^{N}(q_i - q_i^*)^2}}{\sqrt{\sum_{i=1}^{N}(q_i^*)^2}}.$$

The sums include every output frame in the gross-sliding comparison window, $0.15 \leq t \leq 0.70$.

Table 4: Relative errors for $h_0 = 0.5$ in.

Quantity	Abaqus relative error	Coreform IGA relative error
Normal force at $t = 0.1$	39.1%	1.34%
Maximum pressure at $t = 0.1$	66.7%	8.46%
Sliding normal force RMS	27.1%	5.04%
Sliding tangential force RMS	36.6%	4.04%
Sliding maximum pressure RMS	47%	7.06%

Table 5: Relative errors for $h_1 = 0.25$ in.

Quantity	Abaqus relative error	Coreform IGA relative error
Normal force at $t = 0.1$	3.25%	3.45%
Maximum pressure at $t = 0.1$	24.3%	20.1%
Sliding normal force RMS	0.6%	1.58%
Sliding tangential force RMS	5.96%	2.04%
Sliding maximum pressure RMS	25.3%	17%

Table 6: Relative errors for $h_2 = 0.125$ in.

Quantity	Abaqus relative error	Coreform IGA relative error
Normal force at $t = 0.1$	1.07%	3.33%
Maximum pressure at $t = 0.1$	13.4%	8.07%
Sliding normal force RMS	0.376%	1.14%
Sliding tangential force RMS	1.57%	0.604%
Sliding maximum pressure RMS	15.4%	3.68%

7.2 Reaction forces

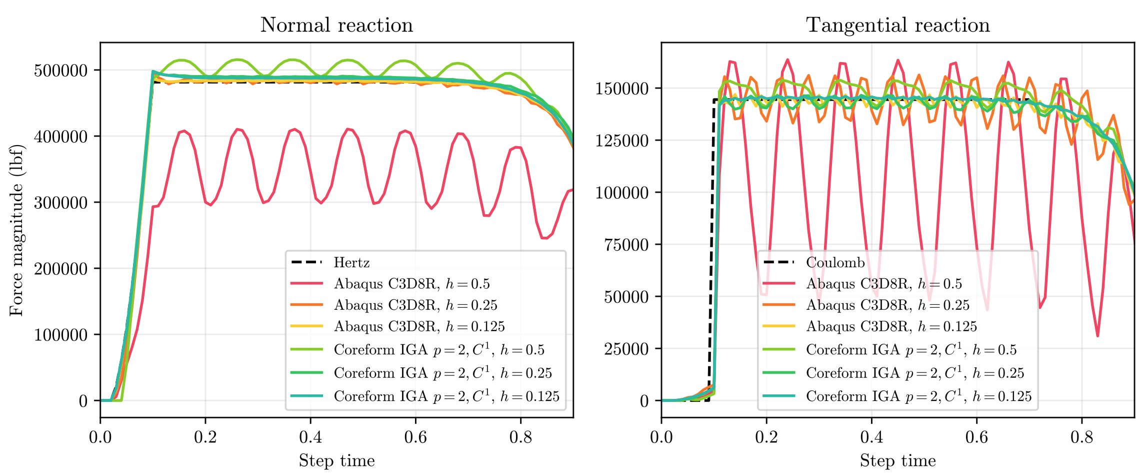


Figure 2: Computed normal and tangential reaction-force magnitudes compared with the Hertz normal force and Coulomb gross-sliding force. Analytical curves stop before edge contact invalidates the half-space model.

7.3 Contact pressure

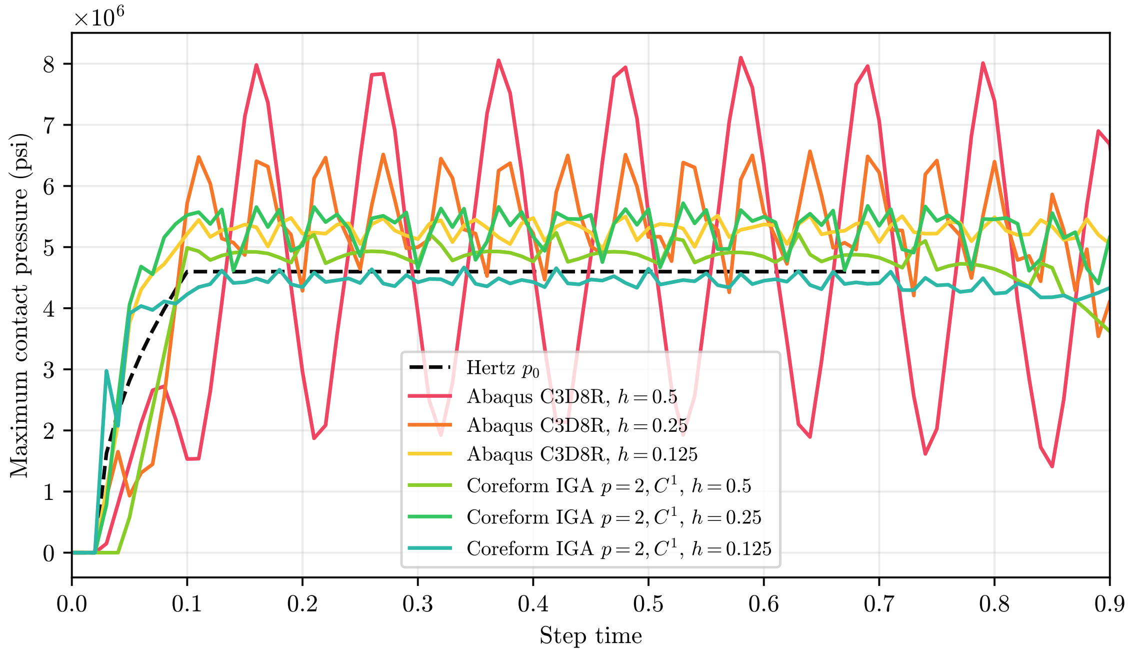
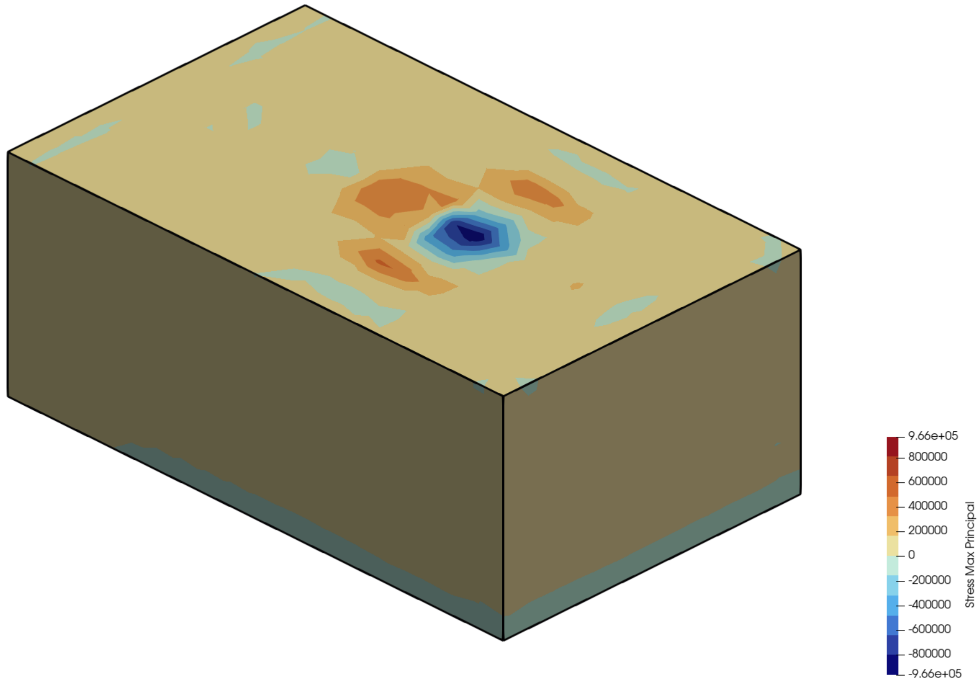


Figure 3: Computed maximum contact pressure compared with the Hertz center pressure. The maximum nodal pressure is especially mesh-sensitive for these coarse cases.

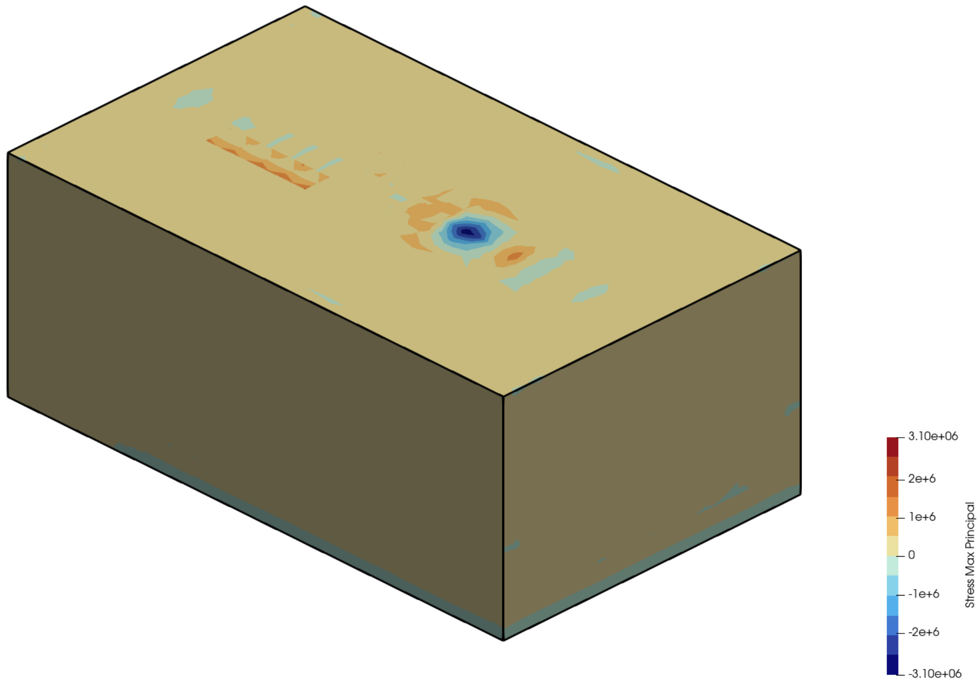
7.4 Maximum principal stress countour plots

The maximum principal stress is shown on the elastic block at $t = 0.55$, during steady sliding. The indenter is omitted to expose the stress field on the block's contact surface.

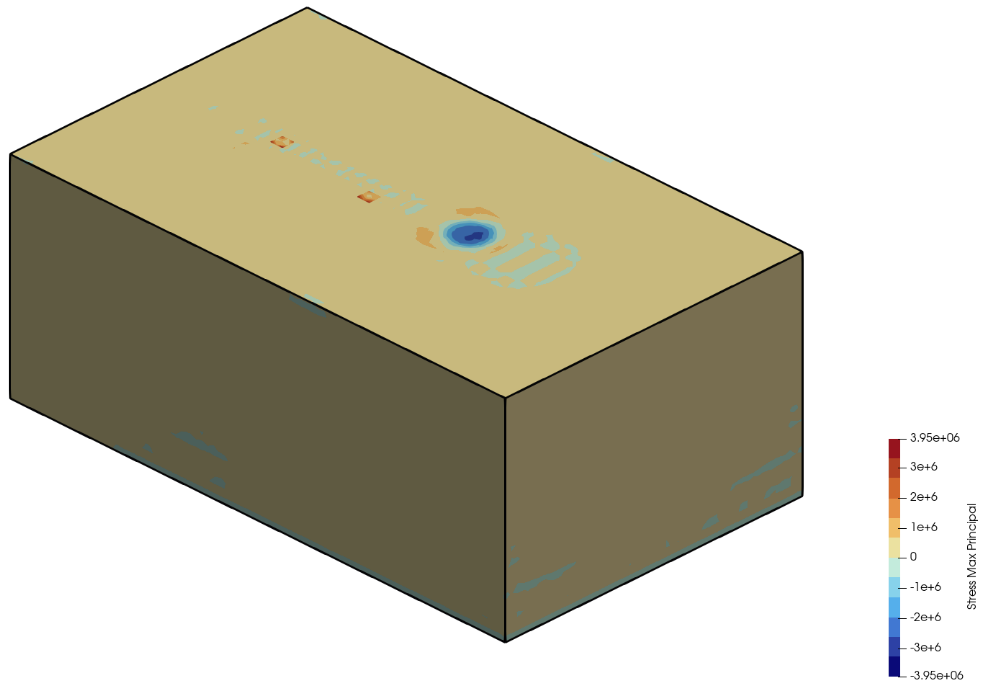
7.4.a $h_0 = 0.5$ **in**



7.4.b $h_1 = 0.25$ **in**



7.4.c $h_2 = 0.125$ in



8 Discussion

The Hertz-Coulomb reference captures both stages of the solution: nonlinear $F_N \propto \delta^{3/2}$ indentation followed by an approximately constant normal force and a friction force near μF_N during translation over the flat surface. The maximum contact pressure result is more sensitive to spatial resolution than the integrated reaction forces because the coarsest mesh does not resolve the contact diameter. The final portion of the simulation is retained as a finite-sliding contact exercise, but it is not used for analytical verification after the contact patch begins interacting with the block edge.

References

[1] Dassault Systèmes, "Coulomb friction." Accessed: Aug. 12, 2026. [Online]. Available: https://docs.software.vt.edu/abaqusv2025/English/SIMACAETHERefMap/simathe-c-coulombfric.htm

[2] K. L. Johnson, *Contact mechanics*. Cambridge University Press, 1985. doi: 10.1017/CBO9781139171731.

[3] V. L. Popov, *Contact mechanics and friction: Physical principles and applications*. Springer, 2010. doi: 10.1007/978-3-642-10803-7.

Appendix

8.1 Capabilities exercised

No qualifying capability evidence was generated for these run artifacts.

8.2 Download artifacts

Download artifacts are available from the HTML version of this page.

8.2.a Geometry CAD files

Geometry CAD downloads are available from the HTML version of this page.

8.2.b Abaqus/CAE journal files

Individual journal downloads are available from the HTML version of this page.

8.3 Supported Abaqus keywords

contact_indenter.inp - outline

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*Heading
*Preprint, echo=NO, model=NO, history=NO, contact=NO
*Node
      1,          2.5,           1.,         -1.5
      2,          2.5,          0.5,         -1.5
      3,          2.5,           0.,         -1.5
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      5,          2.5,          -1.,         -1.5
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      8,           2.,           0.,         -1.5
      9,           2.,         -0.5,         -1.5
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8.4 Provenance

Test_abaqus_verification_contact_indenter_abaqus_h0p1

Parameter	Value
Evidence type	Numerical verification
CTest result	Passed (Release)
Verification result	Passed — 5/5 enforced criteria passed
Test date	26-Aug-2026 17:00:23
Operating system	Ubuntu 26.04 LTS
Coreform IGA version	`2026.8-dev+tewk_1787784671`
Source revision	`bb2d69b00831`
Abaqus version	`Abaqus 6.26-4`

Test_abaqus_verification_contact_indenter_abaqus_h1p1

Parameter	Value
Evidence type	Numerical verification
CTest result	Passed (Release)
Verification result	Passed — 5/5 enforced criteria passed
Test date	26-Aug-2026 17:01:01
Operating system	Ubuntu 26.04 LTS
Coreform IGA version	`2026.8-dev+tewk_1787784671`
Source revision	`bb2d69b00831`
Abaqus version	`Abaqus 6.26-4`

Test_abaqus_verification_contact_indenter_abaqus_h2p1

Parameter	Value
Evidence type	Numerical verification
CTest result	Passed (Release)
Verification result	Passed — 5/5 enforced criteria passed
Test date	26-Aug-2026 17:09:06
Operating system	Ubuntu 26.04 LTS
Coreform IGA version	`2026.8-dev+tewk_1787784671`
Source revision	`bb2d69b00831`
Abaqus version	`Abaqus 6.26-4`

Test_abaqus_verification_contact_indenter_iga_h0p2

Parameter	Value
Evidence type	Numerical verification
CTest result	Passed (Release)
Verification result	Passed — 5/5 enforced criteria passed
Test date	26-Aug-2026 17:01:02
Operating system	Ubuntu 26.04 LTS
Coreform IGA version	`2026.8-dev+tewk_1787784671`
Source revision	`bb2d69b00831`
Abaqus version	`Abaqus 6.26-4`

i Test_abaqus_verification_contact_indenter_iga_h1p2

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Evidence type	Numerical verification
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Verification result	Passed — 5/5 enforced criteria passed
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Operating system	Ubuntu 26.04 LTS
Coreform IGA version	`2026.8-dev+tewk_1787784671`
Source revision	`bb2d69b00831`
Abaqus version	`Abaqus 6.26-4`

i Test_abaqus_verification_contact_indenter_iga_h2p2

Parameter	Value
Evidence type	Numerical verification
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Verification result	Passed — 5/5 enforced criteria passed
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Operating system	Ubuntu 26.04 LTS
Coreform IGA version	`2026.8-dev+tewk_1787784671`
Source revision	`bb2d69b00831`
Abaqus version	`Abaqus 6.26-4`