

Cantilever beam shock and vibration

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Current evidence (exact required source revision
733d3350b4867c5f88715bd09fc1f1409b954da0). Execution status is not itself numerical verification.

1 Objective

Verify mode-based linear dynamics on a cantilever beam: a frequency extraction, a random response to a base-acceleration power spectral density, and a peak response to a base-acceleration shock spectrum.

Coreform IGA elements are Abaqus user elements. Abaqus/Standard will extract eigenmodes for them, but it does not post-process a random response or a response spectrum from those elements, so for an immersed model the frequency extraction is the last result the solver produces. Coreform computes the two remaining procedures from that modal basis instead.

That arrangement has two things to verify, and this study checks both.

1. **The implementation.** On the body-fitted mesh Abaqus computes all three steps itself, so the Coreform random response and response spectrum can be compared against the solver's own answers on an identical modal basis. Any difference here is implementation error alone.
2. **The discretization.** The immersed Coreform IGA modal basis, post-processed by Coreform, is compared against the body-fitted Abaqus analysis end to end. Any difference here is the difference between two meshes.

2 Geometry

Table 1: Cantilever beam geometry.

Parameter	Value
Width b	0.1 m
Height h	0.1 m
Length L	1 m

The beam is the same one used by the static [cantilever beam bending](#) study, so the two chapters describe one physical beam under different analyses. The section is square, which makes every bending mode a degenerate pair and puts a torsional mode inside the analysis band.

3 Material

Table 2: Linear elastic material.

Property	Value
Young's modulus E	$200 \times 10^9 \text{ Pa}$
Poisson's ratio ν	0
Mass density ρ	$8.00 \times 10^3 \text{ kg/m}^3$

Poisson's ratio is zero so that the three-dimensional solid reproduces the one-dimensional beam theory the accepted frequencies come from.

4 Loading & boundary conditions

The root face at $z = 0$ is fully clamped. Both dynamic steps excite the structure through a primary base motion in the global y direction, the weak bending direction of the section.

Table 3: Excitation and analysis settings, shared by both discretizations.

Quantity	Value
Analysis band	10 Hz to 2.00×10^3 Hz
Modes extracted	20
Modal damping ξ	20.0×10^{-3}
Base acceleration PSD	10.0×10^{-3} g ² /Hz, flat
Frequency points per interval	20
Bias parameter	3
Modal summation	SRSS

The random response is driven by a flat base-acceleration spectral density across the band, the shape a vibration qualification test specifies. The response spectrum is driven by a base-acceleration shock spectrum that rises to a plateau and rolls off.

Table 4: Base-acceleration shock spectrum, tabulated at the modal damping ratio.

Magnitude (m/s ²)	Frequency (Hz)
20	10
200	100
200	600
60	2.00×10^3

5 Mesh & discretization

Two discretizations of the same beam. The body-fitted case is a structured C3D8 mesh that Abaqus owns end to end. The immersed case is a Coreform IGA mesh on a rectilinear background grid, trimmed to the beam.

Table 5: Discretizations compared.

Case	Discretization	Element size	Dynamics computed by
<code>abaqus_h0p1</code>	Body-fitted Abaqus (C3D8)	50.0×10^{-3} m	Abaqus/Standard
<code>immersed_h0p1</code>	Immersed Coreform IGA	25.0×10^{-3} m	Coreform, from the modal basis

The immersed case uses a finer background cell than the body-fitted element size. A linear immersed basis needs four cells across the section to resolve bending; at two cells the fundamental frequency comes out several percent stiff.

6 Reference & accepted solutions

The fundamental bending frequency of a clamped-free Euler-Bernoulli beam is

$$f_1 = \frac{\beta_1^2}{2\pi L^2} \sqrt{\frac{EI}{\rho A}}, \quad \beta_1 = 1.8751$$

with $I = bh^3/12$ the second moment of area and $A = bh$ the cross-sectional area. A three-dimensional solid sits slightly above this value because beam theory omits shear flexibility.

The random response and response spectrum quantities have no closed form, so the body-fitted Abaqus analysis is the accepted reference for them. Because Abaqus also computes both procedures on the body-fitted modal basis, the Coreform implementations are additionally checked against the solver directly, which is a stronger statement than the mesh comparison.

7 Results

7.1 Implementation against Abaqus

On a given modal basis the Coreform procedures and Abaqus solve the same problem, so they must agree to solver precision. Abaqus produces displacement-based dynamics output even for user elements, so this comparison is available on both discretizations.

The reported quantity is the component the base motion excites, which is what this chapter's results are about and the only component whose magnitude is set by the excitation rather than by cross-coupling.

Table 6: Coreform against Abaqus/Standard on an identical modal basis, worst relative difference in the excited component. Accepted below 5.00×10^{-6} , and the immersed error is additionally required to stay within 4 times the body-fitted one.

Procedure	Body-fitted	Immersed	Ratio
Random response	212×10^{-9}	213×10^{-9}	1
Response spectrum	97.1×10^{-9}	121×10^{-9}	1.25

Both discretizations agree with the solver at the precision of the single-precision fields Abaqus writes, and they agree with each other: the immersed modal basis costs the implementation nothing. That parity is asserted by the study rather than left to the reader, because it is the claim that lets the immersed workflow stand in for a solver-computed one.

i Why the comparison is per component

Judging the whole displacement field instead would let a small secondary component decide the number. The axial component here peaks near eight percent of the field peak, so points that clear a field-wide floor still sit in that component's own round-off, and the worst of them grows with however many output points a mesh happens to carry. The immersed mesh has roughly eight times as many output points as the body-fitted one, which on a field-wide measure inflates its figure to around 1.4×10^{-6} while the excited component stays at 210×10^{-9} on both. The field-wide numbers are recorded in the downloadable data for transparency, but nothing is gated on them.

7.2 Eigenfrequencies

Table 7: Eigenfrequencies of the two discretizations, with the participation factor in the excitation direction. A mode participates when its factor reaches 1% of the largest.

Mode	Body-fitted (Hz)	Immersed (Hz)	Relative dif- ference	$ \Gamma_y $	Participat- ing
1	81.6	81.8	2.10×10^{-3}	0.116	yes
2	81.6	81.8	2.25×10^{-3}	6.99	yes
3	492	494	5.20×10^{-3}	0.2	yes
4	492	494	5.50×10^{-3}	3.89	yes
5	722	822	0.139	816×10^{-15}	no
6	1.25×10^3	1.25×10^3	292×10^{-6}	31.4×10^{-15}	no
7	1.30×10^3	1.32×10^3	8.46×10^{-3}	9.96×10^{-3}	no
8	1.30×10^3	1.32×10^3	8.75×10^{-3}	2.29	yes

Against the closed form, the body-fitted mesh gives 81.6 Hz and the immersed mesh 81.8 Hz , where Euler-Bernoulli theory predicts 80.8 Hz . Both sit above the beam-theory value, as a shear-flexible solid should.

7.3 Response

Table 8: Peak response in the excitation direction, body-fitted Abaqus against immersed Coreform. Accepted below 50.0×10^{-3} .

Quantity	Body-fitted Abaqus	Immersed Core- form	Relative difference
RMS displacement, random response	$340 \times 10^{-6} m$	$346 \times 10^{-6} m$	17.9×10^{-3}
Peak displacement, shock spectrum	$965 \times 10^{-6} m$	$983 \times 10^{-6} m$	18.3×10^{-3}

8 Discussion

The two halves of the arrangement behave differently, and separating them is the point of this study.

The implementation is exact to solver precision. Given the same modal basis, the Coreform random response and response spectrum reproduce Abaqus to a few parts in ten million, which is the precision at which Abaqus stores its fields. Nothing in the combination introduces error.

The discretizations differ by the amount two meshes differ. The participating eigen-frequencies agree to under one percent, and the RMS and peak responses to under two percent, on a comparison between a body-fitted linear hexahedral mesh and a trimmed immersed spline basis.

Not every mode needs to agree for the response to agree. The square section puts a torsional mode inside the analysis band, and a linear immersed basis represents torsional warping far less accurately than bending: that mode is stiff by an order of ten percent. It carries no participation in a transverse base motion, so it never reaches the response, and the study gates on the participating modes for that reason. The full spectrum is tabulated above so the departure is visible rather than hidden, and it is a useful reminder that agreement mode by mode is neither necessary nor sufficient for agreement in the response.

RMS values depend on the frequency sweep. The random response integrates a spectral density that is sharply peaked at lightly damped resonances, so the reported RMS is only as converged as the sweep beneath it. At the settings used here both codes integrate the same points and agree with each other, but neither is the converged integral. A production run should raise the number of points per interval until the RMS stops moving.

9 References

10 Appendix

10.1 Capabilities exercised

No qualifying capability evidence was generated for these run artifacts.

10.2 Supported Abaqus keywords

i Analysis deck keywords

```

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*Preprint, echo=N0, model=N0, history=N0, contact=N0
*Part, name=<part>
*Node
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  2, 0.100000001, 0.050000007, 1.
  3, 0.100000001, 0., 1.
  4, 0.100000001, 0.100000001, 0.949999988
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 44, 0.100000001, 0.050000007, 0.300000012

```

10.3 Download artifacts

Download artifacts are available from the HTML version of this page.

10.3.a Geometry CAD files

No separate CAD file is required for this problem; the geometry is fully defined in the Abaqus/CAE journal files.

10.3.b Abaqus/CAE journal files

Individual journal downloads are available from the HTML version of this page.

10.4 Provenance

i Test_abaqus_verification_cantilever_shock_vibration_abaqus_hOp1

Parameter	Value
Evidence type	Numerical verification
CTest result	Passed (Release)
Verification result	Passed — 4/4 enforced criteria passed
Test date	30-Sep-2026 16:33:31
Operating system	Ubuntu 22.04.4 LTS
Coreform IGA version	2026.10+70894
Source revision	733d3350b486
Abaqus version	Abaqus Unofficial Packaging Version

i Test_abaqus_verification_cantilever_shock_vibration_immersed_hOp1

Parameter	Value
Evidence type	Numerical verification
CTest result	Passed (Release)
Verification result	Passed — 4/4 enforced criteria passed
Test date	30-Sep-2026 16:33:52
Operating system	Ubuntu 22.04.4 LTS
Coreform IGA version	2026.10+70894
Source revision	733d3350b486
Abaqus version	Abaqus Unofficial Packaging Version