coreform IGA
for Abaqus

Thick-walled spherical pressure vessel

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1 Objective

Verify Coreform IGA for Abaqus on a doubly curved three-dimensional problem whose solution is known exactly: a thick-walled sphere loaded by internal pressure. The Lamé closed-form solution gives the displacement and stress at every point in the structure, so the numerical solution is checked against exact values throughout the three-dimensional domain, and there is no separate reference solve to introduce its own approximation. The quantities of interest are the volume-integrated relative global L^2 errors of the displacement vector, radial displacement, radial stress, and tangential stress.

2 Geometry

One octant of a hollow sphere with inner radius a and outer radius b is shown in Figure 1, with its dimensions provided in Table 1. The octant is oriented so the three cut faces lie on the coordinate planes. By spherical symmetry the octant with symmetry conditions represents the full sphere exactly.

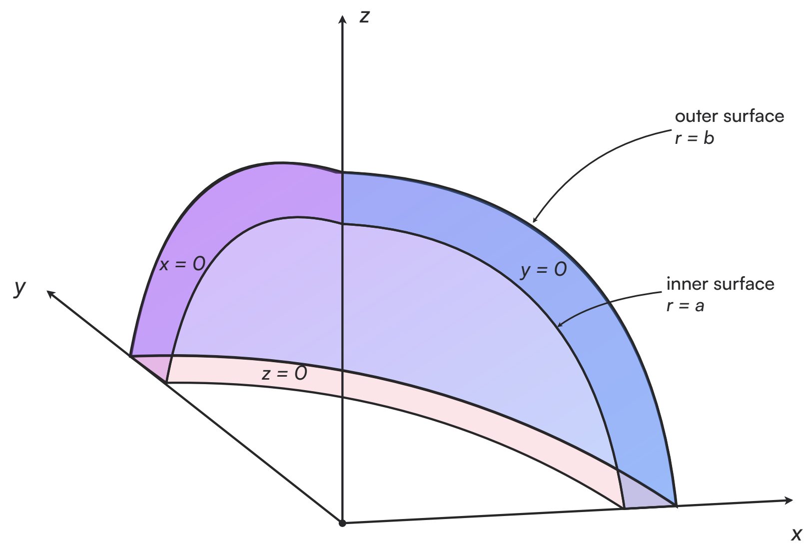


Figure 1: Schematic of the thick-walled spherical shell octant and its coordinate-plane cut faces. The wall thickness is exaggerated for clarity.

Table 1: Geometric parameters.

Quantity	Symbol	Value
Inner radius	a	0.9
Outer radius	b	1
Wall thickness	$b - a$	0.1

3 Material

The sphere is a homogeneous, isotropic, linear-elastic material (Table 2). The model is nondimensional: the modulus and pressure set the scale of the displacement solution but cancel from the relative-error metrics.

Table 2: Material properties.

Property	Symbol	Value
Young's modulus	E	1
Poisson's ratio	ν	0

4 Loading & boundary conditions

A uniform pressure p_i acts on the inner surface; the outer surface is traction-free (Table 3). The three cut faces carry symmetry conditions (zero normal displacement) on the x,

y, and z coordinate planes, which restrain rigid-body motion and complete the octant model of the full sphere.

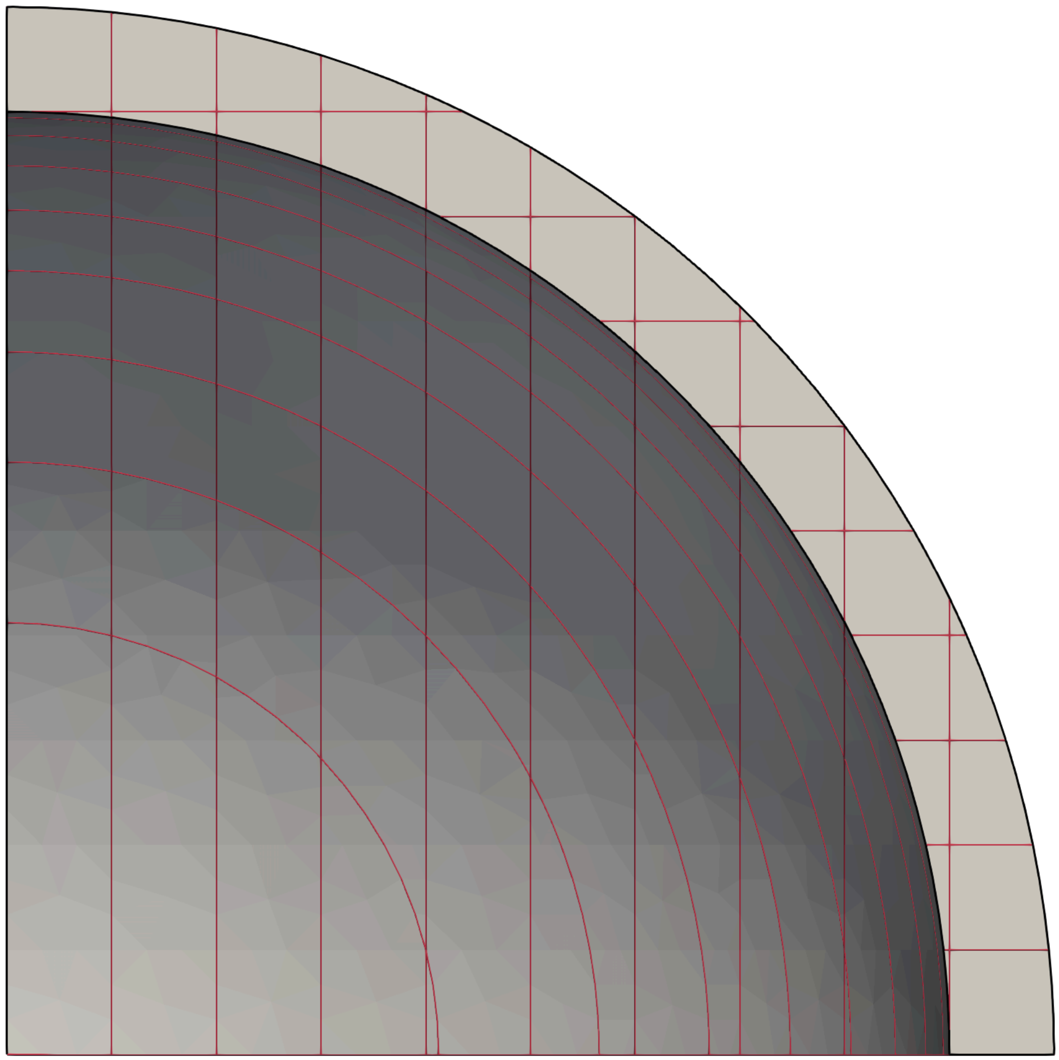
Table 3: Applied loading.

Quantity	Symbol	Value
Inner pressure	p_i	1.00×10^{-6}
Outer pressure	p_o	0

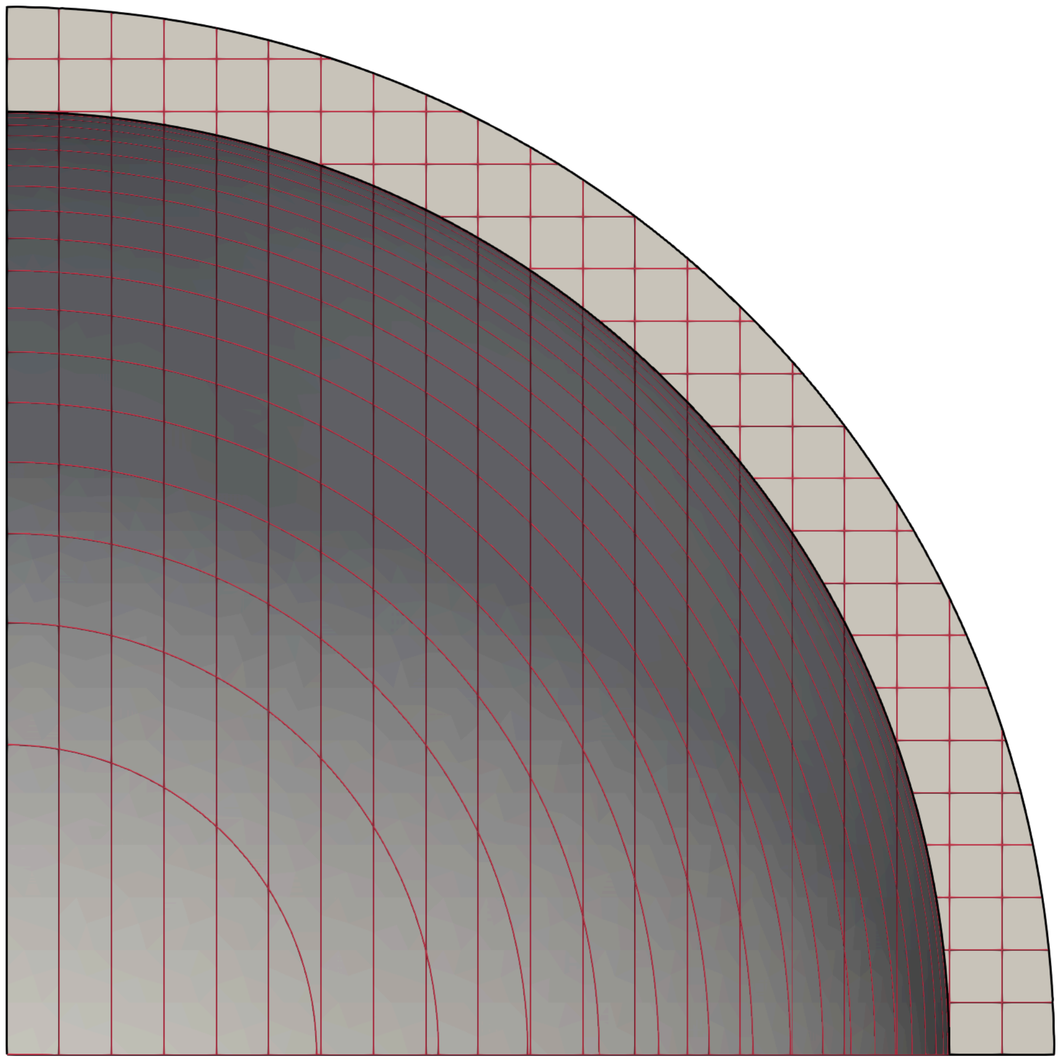
5 Mesh & discretization

Coreform IGA discretizes the octant with immersed, trimmed U-splines: a structured rectilinear background grid is trimmed to the curved shell rather than body-fitting a mesh to it, so the spherical surfaces cut arbitrarily through the grid cells. The background element size is $h = 0.1/2^k$ for refinement levels $k = 0, 1, 2$, at degrees $p = 1$, 2, and 3 with maximum continuity C^{p-1}.

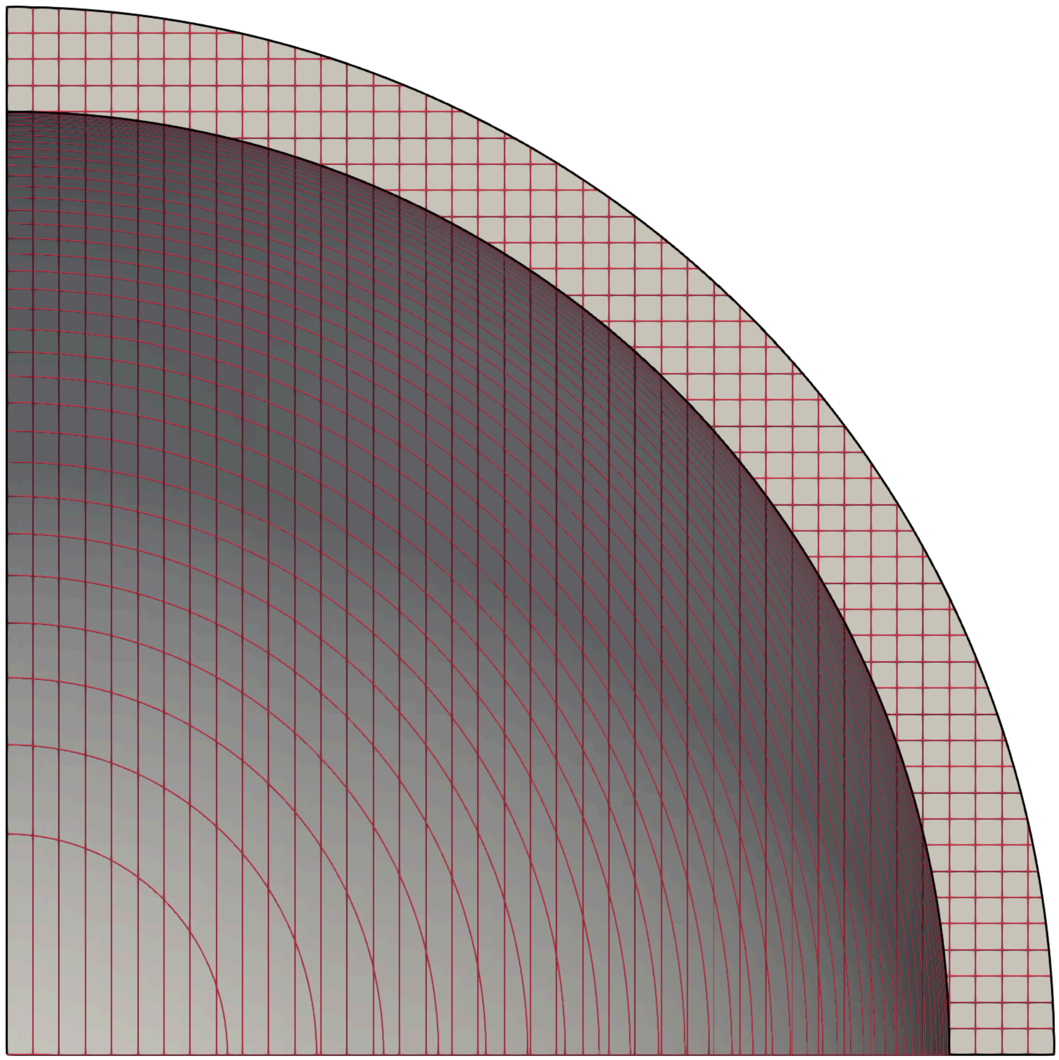
5.0.a h=0.1



5.0.b h=0.05



5.0.c h=0.025



6 Reference & accepted solutions

The accepted solution is the Lamé closed form for a thick-walled sphere under internal and external pressure [1]. With

$$c = \frac{p_i a^3 - p_o b^3}{b^3 - a^3}, \qquad \kappa = \frac{(p_i - p_o)\, a^3 b^3}{b^3 - a^3},$$

the radial and tangential stresses and the radial displacement are

$$\sigma_{rr}(r) = c - \frac{\kappa}{r^3}, \qquad \sigma_{\theta\theta}(r) = c + \frac{\kappa}{2r^3}, \qquad u_r(r) = \frac{1}{E}\Big[(1-2\nu)\, c\, r + (1+\nu)\,\frac{\kappa}{2r^2}\Big].$$

Because the accepted solution is exact, the acceptance criteria bound the true discretization error directly: each refinement level carries its own relative L^2 tolerance per quantity, tightening as the mesh refines (see the tolerance columns in the results table). No separate numerical reference solve is needed.

6.1 Global error evaluation

For this test, a postprocessing routine reads the spline geometry and trimmed quadrature rules from the Coreform IGA source database and reads nodal displacement and integration-point stress from the IGA results database. At every positive-weight trimmed quadrature point, it reconstructs the physical position $\mathbf{x}_q$ and displacement $\mathbf{u}_h(\mathbf{x}_q)$ from the active spline basis. With $\mathbf{n} = \mathbf{x}_q / \|\mathbf{x}_q\|$, it computes

$$u_{r,h} = \mathbf{u}_h \cdot \mathbf{n}, \qquad \sigma_{rr,h} = \mathbf{n}^T \boldsymbol{\sigma}_h \mathbf{n}, \qquad \sigma_{\theta\theta,h} = \frac{\operatorname{tr}\,(\boldsymbol{\sigma}_h) - \sigma_{rr,h}}{2}.$$

For a scalar quantity q, the reported relative global error is

$$\epsilon_q = \frac{\|q_h - q\|_{L^2(\Omega)}}{\|q\|_{L^2(\Omega)}} \approx \left[\frac{\sum_Q w_q \det(\mathbf{J}_q) \, [q_h(\mathbf{x}_q) - q(\mathbf{x}_q)]^2}{\sum_Q w_q \det(\mathbf{J}_q) \, q(\mathbf{x}_q)^2} \right]^{1/2}.$$

The vector-displacement error uses the Euclidean squared norm of $\mathbf{u}_h - \mathbf{u}$ in the numerator and of $\mathbf{u}$ in the denominator. These values are integrated over the volume of the hollow sphere to compute the global L^2 error values for each quantity.

7 Results

7.1 Quantities of interest

For each mesh, the numerical fields for vector displacement, radial displacement, radial stress, and tangential stress are compared against the Lamé closed form in the relative global L^2 norm. The error for each refinement level is shown beside the acceptance tolerance it is required to meet.

7.1.a $p = 1$

h	Vector displacement error	Tolerance	Radial displacement error	Tolerance	Radial stress error	Tolerance	Tangential stress error	Tolerance
0.1	0.585%	0.8%	0.57%	0.8%	28.5%	40%	1.89%	2.5%
0.05	0.199%	0.3%	0.194%	0.3%	15.5%	20%	0.93%	1.2%
0.025	0.0649%	0.1%	0.0632%	0.1%	7.78%	10%	0.437%	0.6%

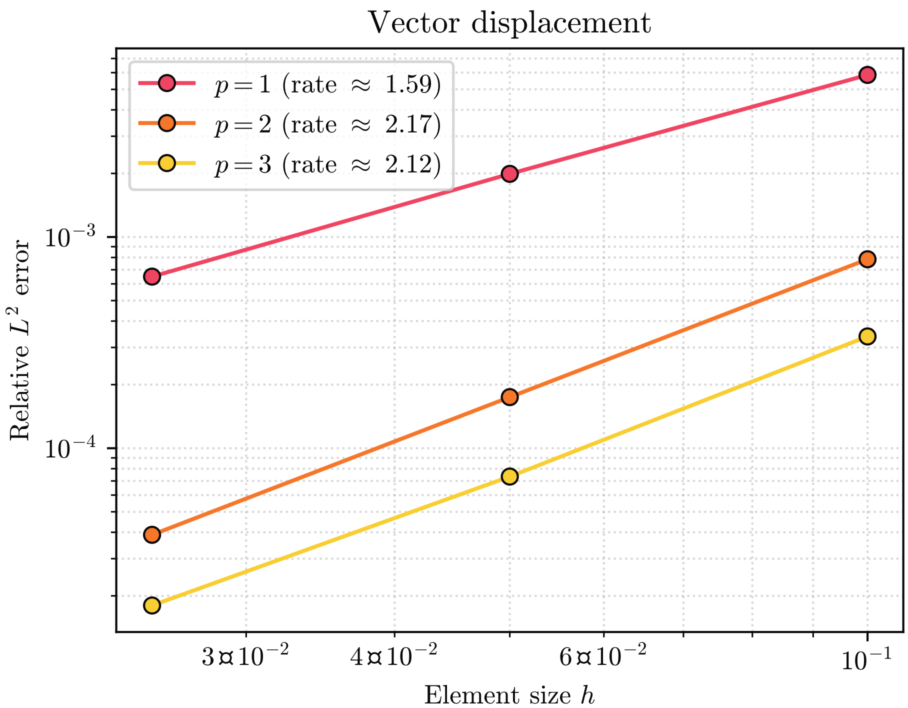
7.1.b $p = 2$

h	Vector displacement error	Tolerance	Radial displacement error	Tolerance	Radial stress error	Tolerance	Tangential stress error	Tolerance
0.1	0.0785%	0.1%	0.0775%	0.1%	1.83%	2.5%	0.146%	0.2%
0.05	0.0175%	0.025%	0.0169%	0.025%	0.659%	0.8%	0.044%	0.06%
0.025	0.00389%	0.005%	0.00338%	0.005%	0.211%	0.3%	0.0137%	0.02%

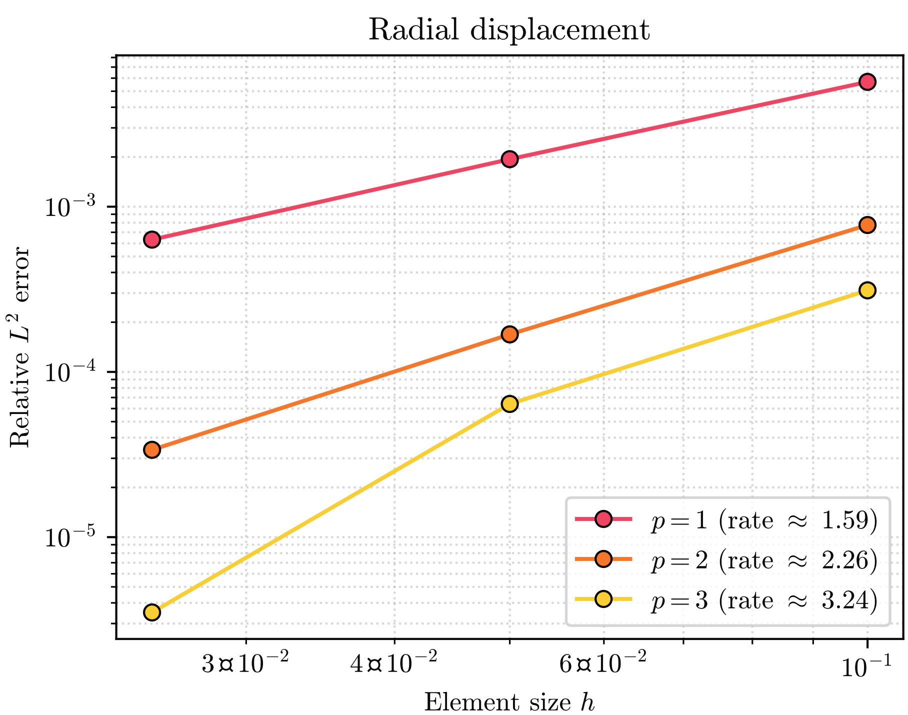
7.1.c $p = 3$

h	Vector displacement error	Tolerance	Radial displacement error	Tolerance	Radial stress error	Tolerance	Tangential stress error	Tolerance
0.1	0.0338%	0.05%	0.0312%	0.04%	0.866%	1.2%	0.0859%	0.12%
0.05	0.00734%	0.01%	0.0064%	0.008%	0.203%	0.3%	0.0174%	0.025%
0.025	0.0018%	0.0025%	$350 \times 10^{-6}\%$	$500 \times 10^{-6}\%$	0.0717%	0.1%	0.00601%	0.008%

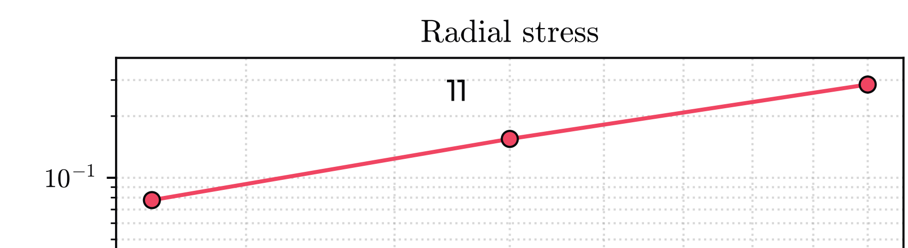
7.2 Convergence



(a) Vector displacement $\mathbf{u}$

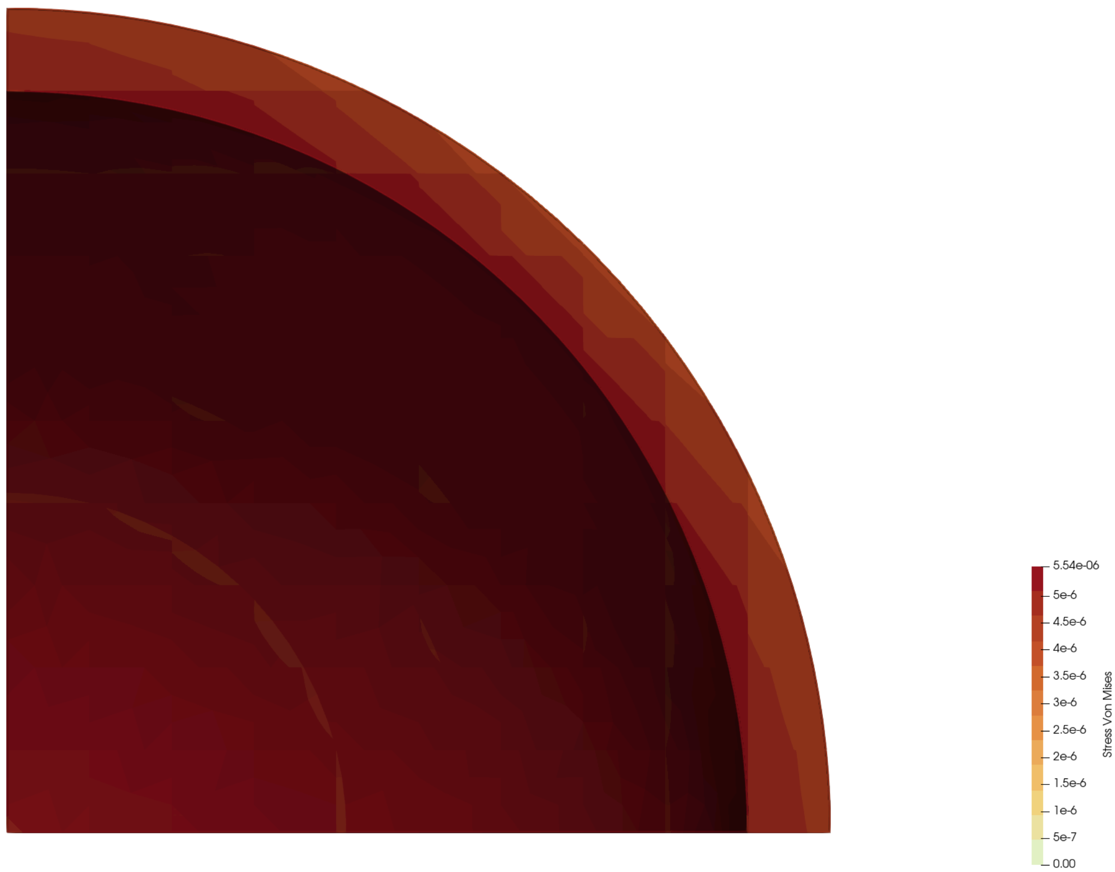


(b) Radial displacement u_r

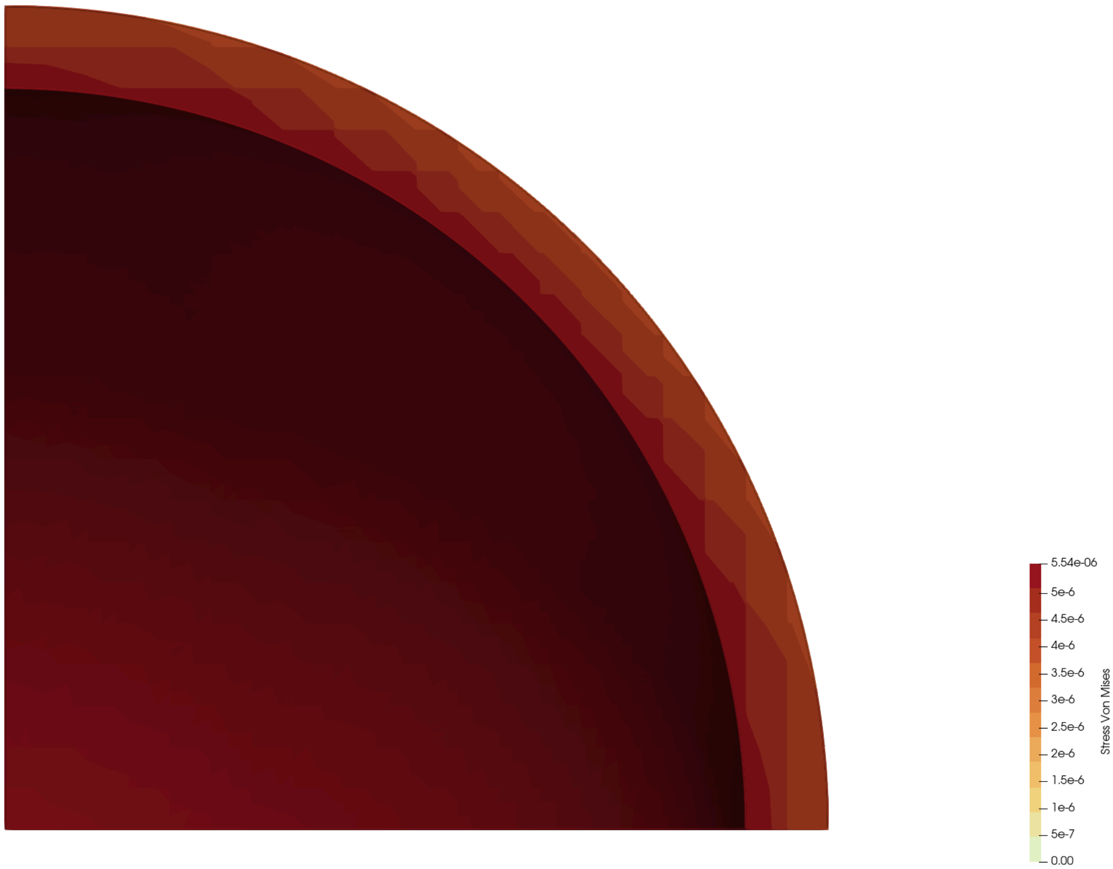


7.3 Contour plots

7.3.a h0p1



7.3.b h1p1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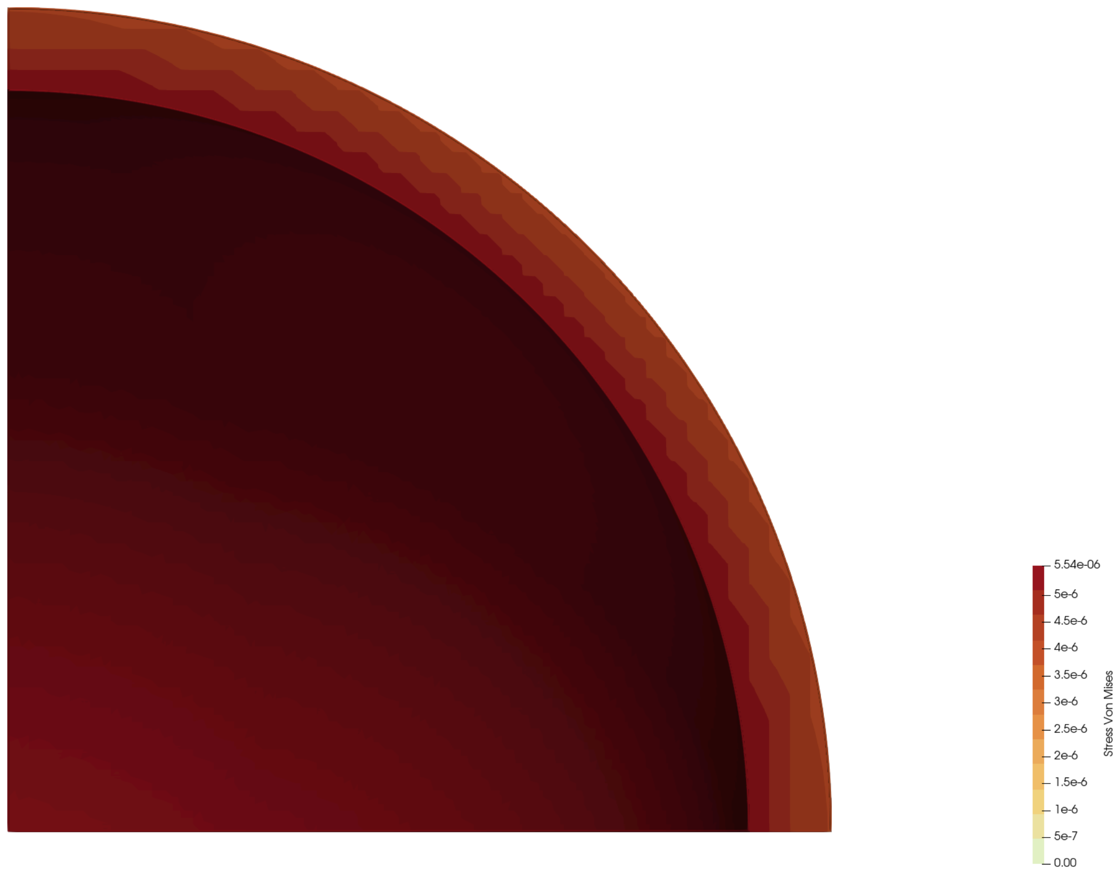
3e-6
2.5e-6
2e-6
1.5e-6
1e-6
5e-7
0.00
Stress Von Mises

7.3.c h2p1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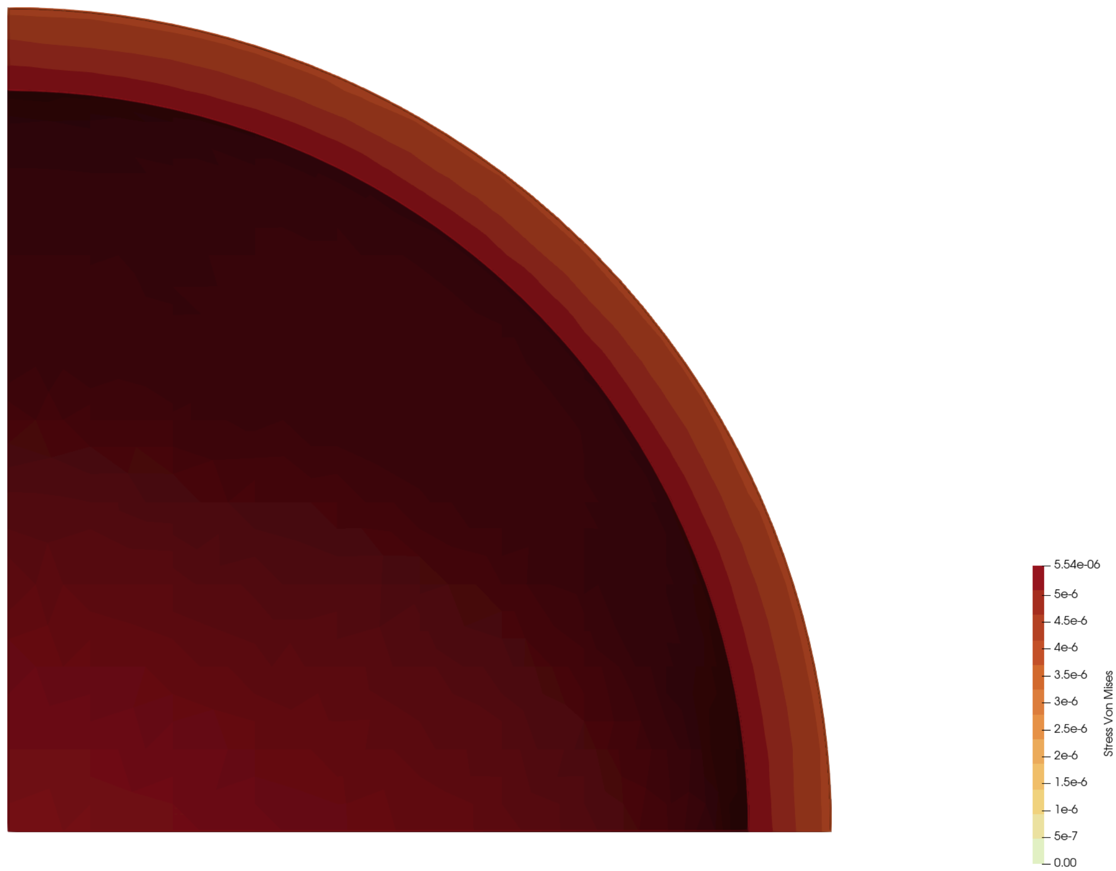
4.5e-6
4e-6
3.5e-6
3e-6
2.5e-6
2e-6
1.5e-6
1e-6
5e-7
0.00
Stress Von Mises

7.3.d h0p2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3.5e-6
3e-6
2.5e-6
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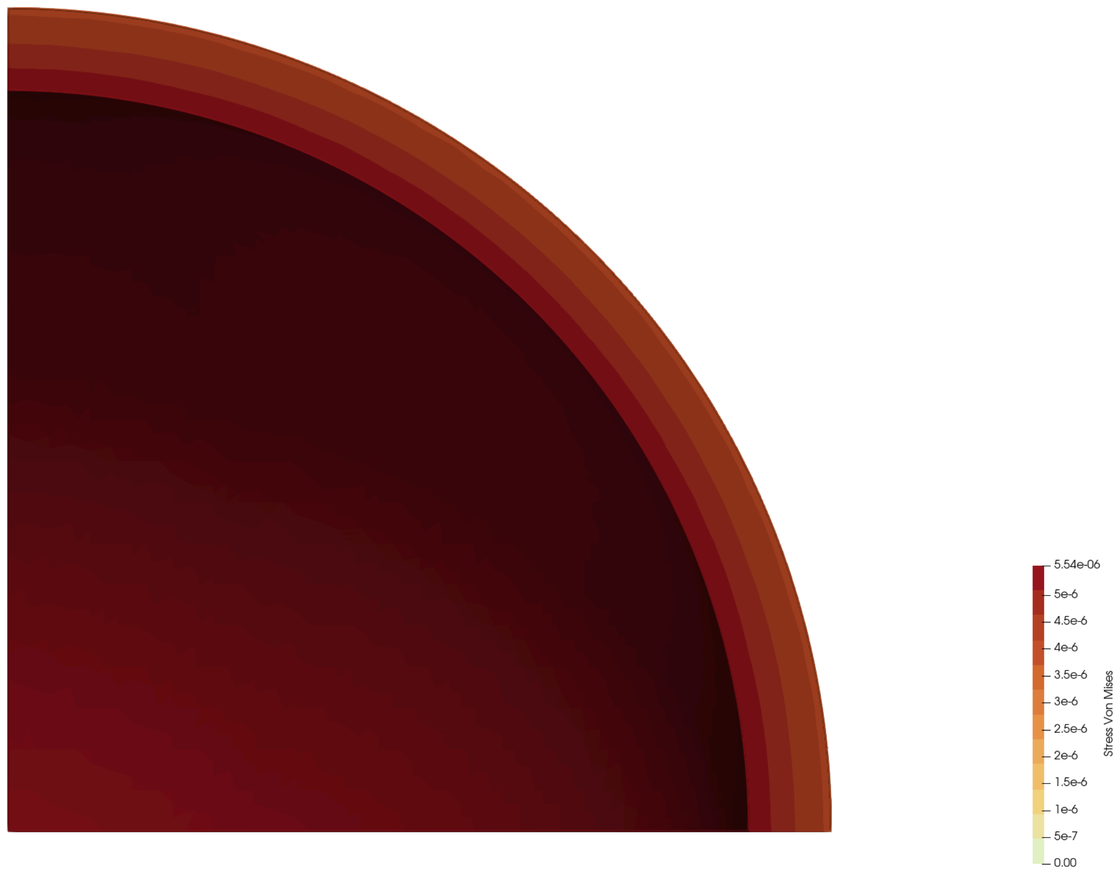
1e-6
5e-7
0.00
Stress Von Mises

7.3.e h1p2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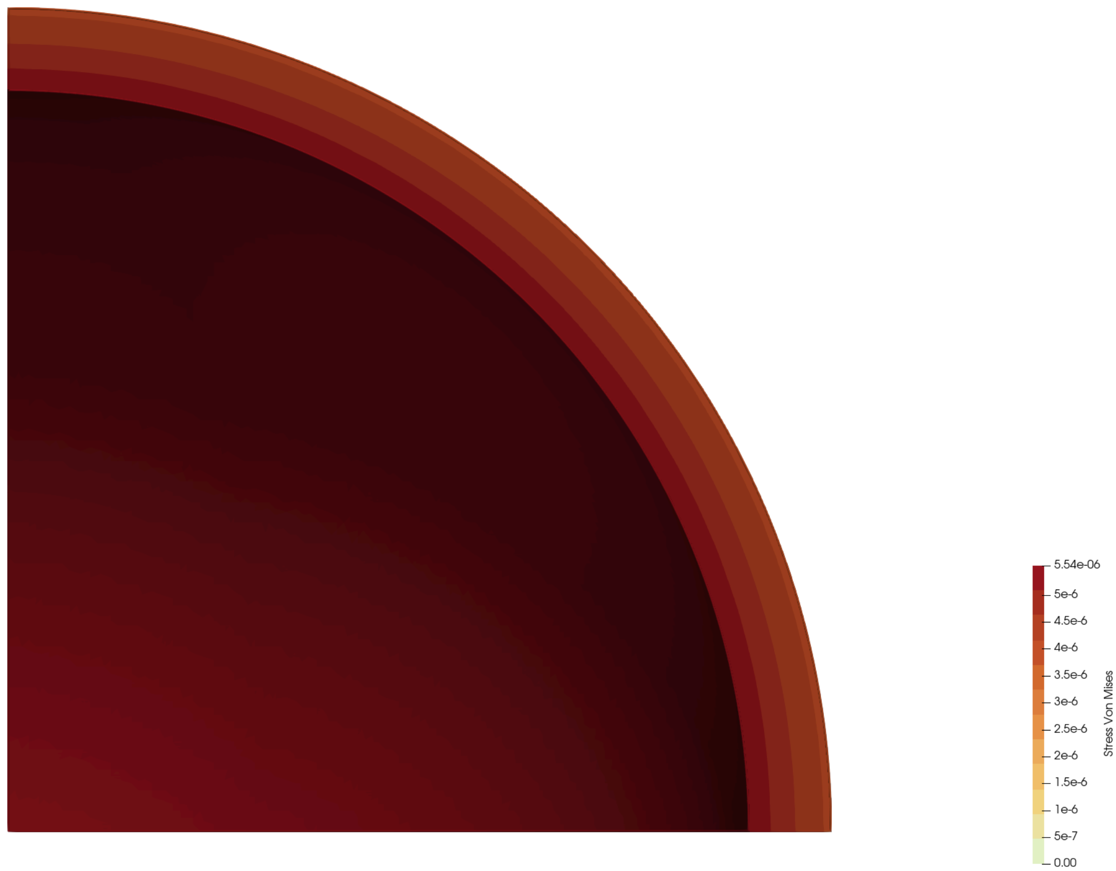
Stress Von Mises

7.3.f h2p2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3e-6
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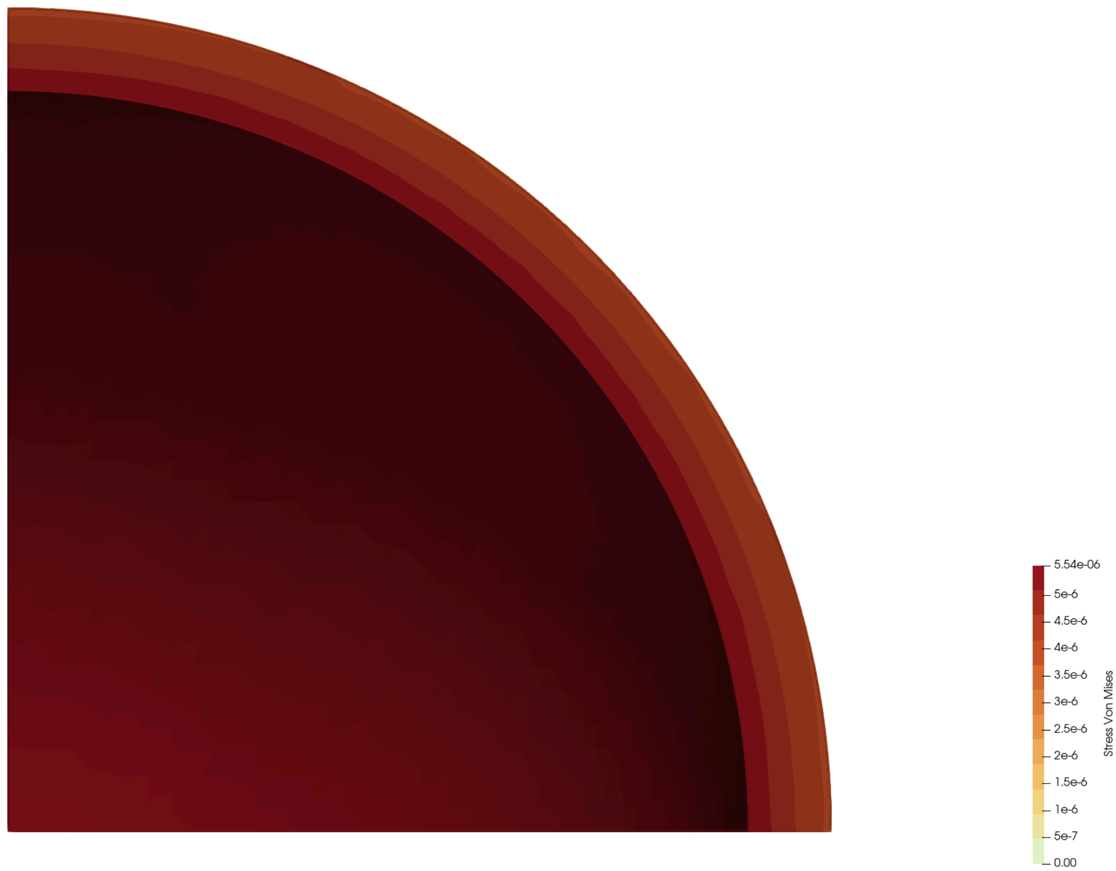
5e-7
0.00
Stress Von Mises

7.3.g h0p3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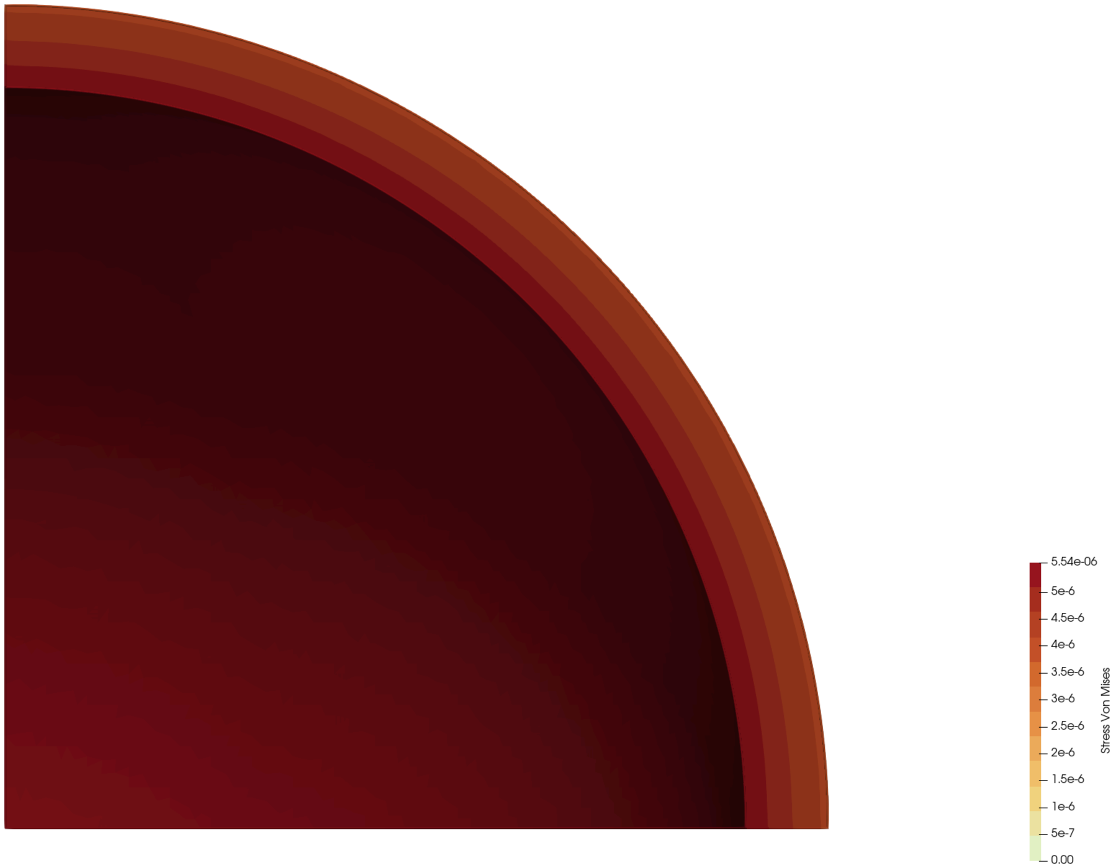
1e-6
5e-7
0.00
Stress Von Mises

7.3.h h1p3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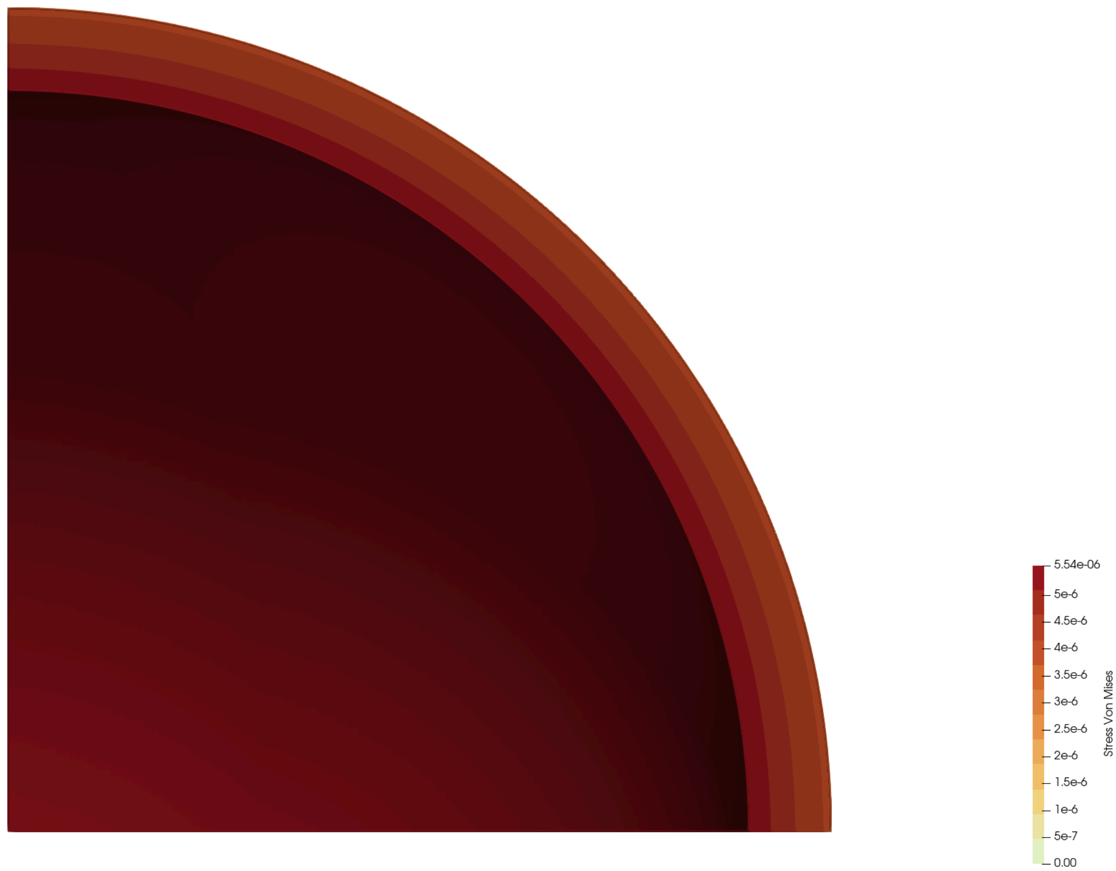
1e-6
5e-7
0.00
Stress Von Mises

7.3.i h2p3



8 Performance

Performance is reported separately for trimming (building the trimmed U-spline basis on the immersing grid) and the Abaqus solve. Abaqus problem size, timing, completion, and actual thread topology come from a sanitized JSON record generated from `abaqus summarize -detail advanced` and the solver message file. The raw summary files can contain host, user, and absolute-path data, so they are not published.

Coreform IGA Mesh

8.0.a $p = 1$

h	Background	Active	Trimmed	CPUs	Wall-time [s]
0.1	$1,728$	343	343	1	11.1
0.05	$10,648$	$1,967$	$1,676$	2	47.6
0.025	$74,088$	$12,460$	$6,785$	4	186.9

8.0.b $p = 2$

h	Background	Active	Trimmed	CPUs	Wall-time [s]
0.1	$2,744$	343	343	1	11.6
0.05	$13,824$	$1,967$	$1,676$	2	47.7
0.025	$85,184$	$12,460$	$6,785$	4	196.6

8.0.c $p = 3$

h	Background	Active	Trimmed	CPUs	Wall-time [s]
0.1	$4,096$	343	343	1	13.4
0.05	$17,576$	$1,967$	$1,676$	2	57.4
0.025	$97,336$	$12,460$	$6,785$	4	243.0

Coreform IGA for Abaqus solve

8.0.d $p = 1$

h	Elements	Nodes	DOFs	Threads	Wall-time [s]
0.1	$2,860$	$2,195$	$6,585$	1	2
0.05	$11,960$	$9,248$	$27,744$	2	2
0.025	$51,093$	$40,234$	$120,702$	4	10

8.0.e $p = 2$

h	Elements	Nodes	DOFs	Threads	Wall-time [s]
0.1	$2,919$	$2,575$	$7,725$	1	2
0.05	$12,918$	$10,904$	$32,712$	2	5
0.025	$56,744$	$47,342$	$142,026$	4	22

8.0.f $p = 3$

h	Elements	Nodes	DOFs	Threads	Wall-time [s]
0.1	$4,622$	$3,930$	$11,790$	1	9
0.05	$19,953$	$15,751$	$47,253$	2	24
0.025	$84,131$	$66,057$	$198,171$	4	99

9 Discussion

The $p = 1$, $p = 2$, and $p = 3$ cases converge monotonically toward the exact solution in all four global quantities. In this problem, the tangential stress is more accurate than the radial stress because it is dominated by the smoothly varying hoop response, while the radial stress must satisfy the traction boundary conditions at both surfaces, which cut arbitrarily through the immersed grid cells. However, these results do demonstrate that a curved geometry with a pressure-loaded surface that does not conform to the background grid can reproduce expected results. We note that the convergence rates do not meet the expected rates from finite element theory, but this is not surprising given the machinery required to integrate immersed IGA into Abaqus/Standard.

References

[1] S. P. Timoshenko and J. N. Goodier, *Theory of elasticity*, 3rd ed. New York: McGraw-Hill, 1970.

Appendix

9.1 Capabilities exercised

The table is generated from current passing simulation and study artifacts. Each row must satisfy the linked capability card's evidence contract; declaring a capability in a test does not, by itself, create evidence.

Table 4: Capabilities exercised by the current verification artifacts.

Capability	Evidence level	Official verification test
Linear static analysis	Numerically validated	Test_abaqus_verification_pressurized_sphere_immersed_h2p2
Cartesian symmetry boundary conditions	Numerically validated	Test_abaqus_verification_pressurized_sphere_immersed_h2p2
Threaded Abaqus/Standard execution	Numerically validated	Test_abaqus_verification_pressurized_sphere_immersed_h2p2
Parallel Coreform IGA mesh generation	Numerically validated	Test_abaqus_verification_pressurized_sphere_immersed_h2p2
Uniform pressure loading	Numerically validated	Test_abaqus_verification_pressurized_sphere_immersed_h2p2
Linear elastic material behavior	Numerically validated	Test_abaqus_verification_pressurized_sphere_immersed_h2p2
Linear C0 spline basis	Numerically validated	Test_abaqus_verification_pressurized_sphere_immersed_h2p1
Quadratic C1 spline basis	Numerically validated	Test_abaqus_verification_pressurized_sphere_immersed_h2p2
Cubic C2 spline basis	Numerically validated	Test_abaqus_verification_pressurized_sphere_immersed_h2p3
Immersed rectilinear meshes	Numerically validated	Test_abaqus_verification_pressurized_sphere_immersed_h2p2
Explicit field-output selection	Numerically validated	Test_abaqus_verification_pressurized_sphere_immersed_h2p2
Stress field output (S)	Exercised	Test_abaqus_verification_pressurized_sphere_immersed_h2p2
Displacement field output (U)	Numerically validated	Test_abaqus_verification_pressurized_sphere_immersed_h2p2
Dependent part instances	Numerically validated	Test_abaqus_verification_pressurized_sphere_immersed_h2p2

9.2 Download artifacts

Download artifacts are available from the HTML version of this page.

9.2.a Geometry CAD files

No separate CAD file is required for this problem; the geometry is fully defined in the Abaqus/CAE journal files.

9.2.b Abaqus/CAE journal files

Individual journal downloads are available from the HTML version of this page.

9.3 Supported Abaqus keywords

The deck below is reproduced from the verification test's Abaqus input file with comments stripped and instance-specific names (sets, materials, files) redacted to `<placeholders>`.

pressurized_sphere_h0.inp - outline

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*Heading
*Preprint, echo=NO, model=NO, history=NO, contact=NO
*Include, input=<iga-mesh-inp>
*System
*Material, name=<material>
*Elastic
1.,0.
*User Output Variables
1,
*Boundary
<node-set>, XSYMM
*Boundary
<node-set>, YSYMM
*Boundary
<node-set>, ZSYMM
*Step, name=<step>, nlgeom=NO, inc=1
*Static, direct
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*Restart, write, frequency=0
*Output, field
*Node Output
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*Element Output, position=AVERAGED AT NODES, directions=YES
S,
*Output, history, variable=PRESELECT
*End Step
```

9.4 Provenance

The records below identify the execution environment and verification basis for every CTest artifact reported on this page.

i Test_abaqus_verification_pressurized_sphere_immersed_h0p1

Parameter	Value
Evidence type	Numerical verification
CTest result	Passed (Release)
Verification result	Passed — 4/4 enforced criteria passed
Test date	27-Jul-2026 20:34:15
Operating system	Ubuntu Linux 24.04.2 LTS (Noble Numbat)
Coreform IGA version	`2026.7-dev+gvernon2_1785204663`
Source revision	`db258ffe9358`
Abaqus version	`2026_06_24-12.44.52 DEVr428 214170`

i Test_abaqus_verification_pressurized_sphere_immersed_h1p1

Parameter	Value
Evidence type	Numerical verification
CTest result	Passed (Release)
Verification result	Passed — 4/4 enforced criteria passed
Test date	27-Jul-2026 20:39:56
Operating system	Ubuntu Linux 24.04.2 LTS (Noble Numbat)
Coreform IGA version	`2026.7-dev+gvernon2_1785204663`
Source revision	`db258ffe9358`
Abaqus version	`2026_06_24-12.44.52 DEVr428 214170`

i Test_abaqus_verification_pressurized_sphere_immersed_h2p1

Parameter	Value
Evidence type	Numerical verification
CTest result	Passed (Release)
Verification result	Passed — 4/4 enforced criteria passed
Test date	27-Jul-2026 20:48:06
Operating system	Ubuntu Linux 24.04.2 LTS (Noble Numbat)
Coreform IGA version	`2026.7-dev+gvernon2_1785204663`
Source revision	`db258ffe9358`
Abaqus version	`2026_06_24-12.44.52 DEVr428 214170`

i Test_abaqus_verification_pressurized_sphere_immersed_h0p2

Parameter	Value
Evidence type	Numerical verification
CTest result	Passed (Release)
Verification result	Passed — 4/4 enforced criteria passed
Test date	27-Jul-2026 20:34:22
Operating system	Ubuntu Linux 24.04.2 LTS (Noble Numbat)
Coreform IGA version	`2026.7-dev+gvernon2_1785204663`
Source revision	`db258ffe9358`
Abaqus version	`2026_06_24-12.44.52 DEVr428 214170`

ℹ Test_abaqus_verification_pressurized_sphere_immersed_h1p2

Parameter	Value
Evidence type	Numerical verification
CTest result	Passed (Release)
Verification result	Passed — 4/4 enforced criteria passed
Test date	27-Jul-2026 20:40:06
Operating system	Ubuntu Linux 24.04.2 LTS (Noble Numbat)
Coreform IGA version	`2026.7-dev+gvernon2_1785204663`
Source revision	`db258ffe9358`
Abaqus version	`2026_06_24-12.44.52 DEVr428 214170`

ℹ Test_abaqus_verification_pressurized_sphere_immersed_h2p2

Parameter	Value
Evidence type	Numerical verification
CTest result	Passed (Release)
Verification result	Passed — 4/4 enforced criteria passed
Test date	27-Jul-2026 20:48:54
Operating system	Ubuntu Linux 24.04.2 LTS (Noble Numbat)
Coreform IGA version	`2026.7-dev+gvernon2_1785204663`
Source revision	`db258ffe9358`
Abaqus version	`2026_06_24-12.44.52 DEVr428 214170`

i Test_abaqus_verification_pressurized_sphere_immersed_h0p3

Parameter	Value
Evidence type	Numerical verification
CTest result	Passed (Release)
Verification result	Passed — 4/4 enforced criteria passed
Test date	27-Jul-2026 20:34:37
Operating system	Ubuntu Linux 24.04.2 LTS (Noble Numbat)
Coreform IGA version	`2026.7-dev+gvernon2_1785204663`
Source revision	`db258ffe9358`
Abaqus version	`2026_06_24-12.44.52 DEVr428 214170`

i Test_abaqus_verification_pressurized_sphere_immersed_h1p3

Parameter	Value
Evidence type	Numerical verification
CTest result	Passed (Release)
Verification result	Passed — 4/4 enforced criteria passed
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Source revision	`db258ffe9358`
Abaqus version	`2026_06_24-12.44.52 DEVr428 214170`

i Test_abaqus_verification_pressurized_sphere_immersed_h2p3

Parameter	Value
Evidence type	Numerical verification
CTest result	Passed (Release)
Verification result	Passed — 4/4 enforced criteria passed
Test date	27-Jul-2026 20:53:17
Operating system	Ubuntu Linux 24.04.2 LTS (Noble Numbat)
Coreform IGA version	`2026.7-dev+gvernon2_1785204663`
Source revision	`db258ffe9358`
Abaqus version	`2026_06_24-12.44.52 DEVr428 214170`

i Test_abaqus_verification_study_pressurized_sphere_immersed_p1

Parameter	Value
Evidence type	Numerical verification
CTest result	Passed (Release)
Verification result	Passed — 4/4 enforced criteria passed
Test date	28-Jul-2026 03:09:22 UTC
Operating system	—
Coreform IGA version	Not applicable — postprocessing-only CTest
Source revision	`db258ffe9358`
Abaqus version	Not applicable — postprocessing-only CTest

i Test_abaqus_verification_study_pressurized_sphere_immersed_p2

Parameter	Value
Evidence type	Numerical verification
CTest result	Passed (Release)
Verification result	Passed — 4/4 enforced criteria passed
Test date	28-Jul-2026 03:09:22 UTC
Operating system	—
Coreform IGA version	Not applicable — postprocessing-only CTest
Source revision	`db258ffe9358`
Abaqus version	Not applicable — postprocessing-only CTest

i Test_abaqus_verification_study_pressurized_sphere_immersed_p3

Parameter	Value
Evidence type	Numerical verification
CTest result	Passed (Release)
Verification result	Passed — 4/4 enforced criteria passed
Test date	28-Jul-2026 03:09:22 UTC
Operating system	—
Coreform IGA version	Not applicable — postprocessing-only CTest
Source revision	`db258ffe9358`
Abaqus version	Not applicable — postprocessing-only CTest